

Mixed-Mode Fracture Criterion of Short Carbon Fiber-Dispersed SiC Matrix Composite

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Abstract

Fracture toughness tests for short carbon fiber-dispersed SiC matrix (SCF/SiC) composites fabricated by means of the silicon melt infiltration (MI) process have been carried out under mode I, mode II and mixed I/II mode loading conditions. A Brazilian disk specimen was used for the mixed-mode fracture toughness test. The results showed that the fracture toughness under mixed I/II mixed-mode loading conditions was substantially higher than that under mode I loading. The relationship between mixed-mode fracture toughness and the mode mixity parameter was found to be in agreement with the generalized maximum tangential stress (G-MTS) criterion. Compressive T-stress acting parallel to the notch direction plays an important role in affecting the mixed-mode fracture toughness of SCF/SiC composite using Brazilian disk specimens.

Keywords: Composite, carbon fiber, SiC, toughness, mixed mode

I. Introduction

Short carbon fiber-dispersed SiC matrix (hereafter, denoted as SCF/SiC) composites are used for high-wear component applications, e.g., as brake disks for high-performance cars and emergency brake pads for high-speed trains and elevators ^{1–5}. The mechanical performance parameters of SCF/SiC composites, including their Young's modulus, bending strength, compressive strength, and wear properties, have been characterized extensively. However, studies on the fracture behavior of SCF/SiC composites are limited ^{6, 7}. Inoue et al. recently reported that the mode I fracture toughness of the SCF/SiC composite is $K_{Ic} \sim 3.5 - 4.1 \text{ MPa}\sqrt{\text{m}}$ ^{6, 7}. This fracture toughness value is quite low, similar to that of monolithic SiC.

In the above-mentioned practical applications, stress concentration sources, such as micro-cracks from mismatches in the thermal expansion coefficients of SCF and SiC, pores, and surface flaws that exist in SCF/SiC composites, may be subjected to a combination of tensile and shear loading during service. The safe design of SCF/SiC composites for potential structural applications inevitably also requires the collection of fracture data and determination of fracture criteria under different loading modes.

The objective of this study is to examine the fracture behavior and to evaluate the fracture toughness of SCF/

SiC composites under I+II mixed-mode loading and pure mode II loading conditions. Special attention is given to the fracture behavior near stress concentration sources. The experimental results are compared with mixed-mode fracture criteria proposed in the literature. Moreover, the influence of T-stress on the relationship between fracture toughness and the mode mixity parameter is examined, and a mixed-mode fracture criterion for SCF/SiC is discussed.

II. Experimental procedure

(1) Composite material

SCF/SiC composites fabricated by means of a Si melt infiltration (MI) process were used in this study. The as-fabricated disk-shaped composite was obtained from Covale Material Co., Ltd. (Tokyo, Japan). Pitch-based short carbon fiber (DIALEAD®, K-223HG, Mitsubishi Plastics Inc., Tokyo, Japan) was used as reinforcement. A typical polished in-plane section of the SCF/SiC composite is shown in Fig. 1a with the macroscopic appearance of the as-received disk specimen. Short carbon fibers are dispersed in units of bundles (hereafter denoted as “SCF/SiC mini-composite”). The distribution of the SCF/SiC mini-composites is random in the plane ⁶. Fig. 1b shows the typical microstructure of SCF/SiC composite. Microcracks are observed in both the Si and SiC phases. The microcracks in the composite range in length from $\sim 200 \mu\text{m}$ to

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~1400 μm . Selected mechanical properties of the SCF/SiC composite are listed in Table 1^{6,7}.

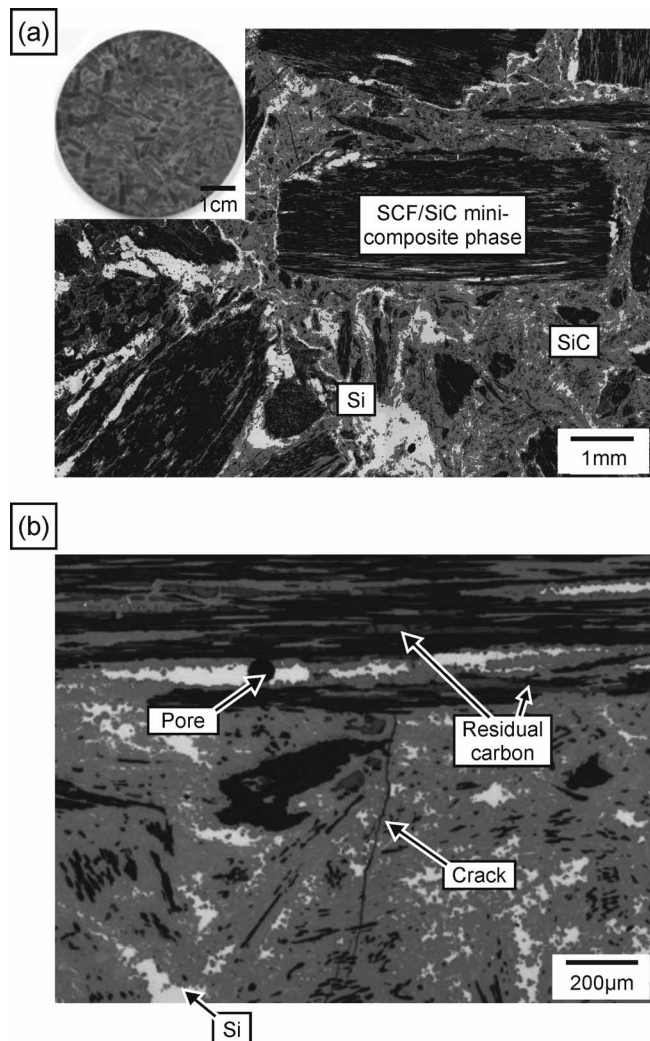


Fig. 1: Typical example of (a) polished in-plane section with macroscopic appearance of disk specimen, and (b) microcrack.

Table 1: Some mechanical properties of SCF/SiC composite.

Properties	
Young's modulus (Tensile load) (GPa)	35–50 (In-plane)
Tensile strength (MPa)	5–10 (In-plane)
Young's modulus (Compressive load) (GPa)	60–70 (In-plane) 40–55 (Through-the-thickness plane)
Compressive strength (MPa)	220–240 (In-plane) 180–200 (Through-the-thickness plane)
Poisson's ratio*	0.2

*Poisson's ratio is obtained from uni-axial compression test along xy -plane. Strain gauges are affixed to cylinder shape specimen surface parallel and perpendicular to compressive loading direction. Poisson's ratio (ν) is given by $\nu = -\epsilon_L/\epsilon_A$. Here, ϵ_L and ϵ_A are strain along longitudinal direction and circumferential strain, respectively.

(2) Test procedure

Fig. 2a shows the shape and dimensions of the disk specimens used in this study. The diameter ($2R$) and thickness (t) of the disk specimen are 50 mm and 3 mm, respectively. The parallel flat surfaces of the disk were carefully polished in a standard metallurgical procedure, utilizing a 0.5- μm diamond slurry for finishing. After the final polishing step, a straight notch was introduced to the center of a specimen using an arc-discharge machining process. The width of the notch was ~100 μm and the radius of the notch tip was ~75 μm . The initial notch length in the disk specimen ($2a_0$) was ~25 mm, which corresponded to the notch length radius ratio $a_0/R \sim 0.5$. Hereafter, this composite is called the “Brazilian disk” specimen.

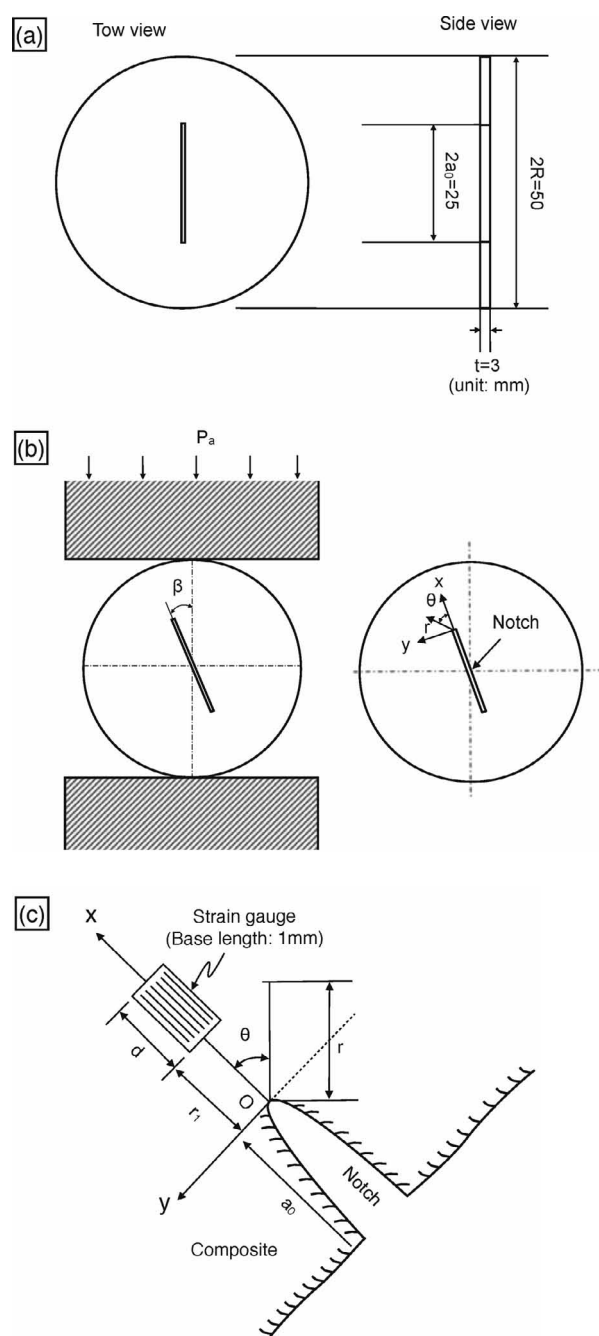


Fig. 2: (a) Shape and dimensions of Brazilian disk specimen, (b) schematic of test procedure, with the definitions of the x - y and

r - θ axes and the dimensions of this research, and (c) position and direction used to determine the T -stress using strain gauge.

Fracture toughness testing of the SCF/SiC composite under pure mode I, pure mode II, and I+II mixed-mode loading conditions was performed using Brazilian disk specimens. Fig. 2b shows a schematic of the loading configuration, with the x - y and r - θ coordinates defined. A uniaxial compressive load was applied to the specimen using a screw-driven test machine (maximum capacity 50 kN, Model 4204, Instron Corp., MA, USA) with a constant crosshead displacement rate of 5×10^{-5} m/min until the specimen completely failed.

A wire-wound strain gauge (gauge length $d \sim 1.0$ mm, KFG-2-120-C1-11LIM2R, Kyowa Electronic Instrument Co., Ltd., Tokyo) was affixed ahead of the notch tip using cyanoacrylate adhesive. The strain measurement direction was parallel to the notch direction (x -direction) (Fig. 2c). This strain was defined as $\bar{\epsilon}_x$. The applied compressive load P_a , crosshead displacement u , and strain $\bar{\epsilon}_x$ were continuously stored to a data collection system (NR-500, Keyence Corp., Osaka, Japan) at a sampling step time of 50 ms. The fracture toughness tests were carried out in ambient conditions (temperature $\sim 23 \pm 1$ °C, relative humidity $\sim 50 \pm 10$ %).

Fig. 2b shows the loading procedures for the Brazilian disk specimen. In order to change the loading mode, the notch direction is aligned at an angle β to the loading direction. The loading angle β had a range of 0 to ~ 23 degrees; the β value of 23 degrees corresponds to the pure mode II loading condition. These experimental parameters were determined based on the previous study⁸. The accuracy of the loading angle β in this experiment was within ~ 0.5 degrees.

The mode I and mode II critical stress intensity factors under single-mode loading, $K_I^c(\beta)$ and $K_{II}^c(\beta)$, respectively, are calculated using⁸:

$$K_I^c(\beta) = \frac{P_{\max} \sqrt{\pi a_0}}{\pi R t} Y_I \left(\frac{a_0}{R}, \beta \right), \quad (1)$$

and

$$K_{II}^c(\beta) = \frac{P_{\max} \sqrt{\pi a_0}}{\pi R t} Y_{II} \left(\frac{a_0}{R}, \beta \right), \quad (2)$$

where $P_{\max}$ is the maximum load. For calculation, the maximum load $P_{\max}$ is used because unstable crack propagation, which is independent of the loading mode, occurs at this load. In Equations (1) and (2), $Y_I(a_0/R, \beta)$ and $Y_{II}(a_0/R, \beta)$ depend on the ratio of the crack length to the radius of specimen a_0/R and the loading angle β . They are dimensionless coefficients, given by⁸:

$$Y_I = \sum_{i=1}^n T_i \left(\frac{a_0}{R} \right)^{2i-2} A_i(\beta), \quad (3)$$

and

$$Y_{II} = 2 \sin 2\beta \sum_{i=1}^n S_i \left(\frac{a_0}{R} \right)^{2i-2} B_i(\beta). \quad (4)$$

Here, A_i , B_i , S_i , and T_i for $a_0/R = 0.5$ are given in Table A-1 and A-2⁸.

The effective stress intensity factor at the maximum load and loading angle β , $K_{\text{eff}}^c(\beta)$, is given by⁹:

$$K_{\text{eff}}^c(\beta) = \sqrt{[K_I^c(\beta)]^2 + [K_{II}^c(\beta)]^2}, \quad (5)$$

and the mode mixity parameter, M_e , is defined as:

$$M_e = \frac{2}{\pi} \tan^{-1} \left[\frac{K_I^c(\beta)}{K_{II}^c(\beta)} \right]. \quad (6)$$

The value of M_e varies from $M_e = 0$ for pure mode II loading to $M_e = 1$ for pure mode I loading.

During the fracture toughness test, direct observation at the notch tip was performed using a high-resolution charge-coupled device (CCD) camera (VH-Z100UR, KEYENCE Co., Osaka, Japan). Microscopic observation was performed from the notch tip to $r \sim 5$ mm with a width of ~ 5 mm. After this, the specimens were observed by means of optical microscopy (Axioplan2 imaging, Carl Zeiss Corp., Jena, Germany) and scanning electron microscopy (SEM, TM3000, Hitachi High Technologies Corp., Tokyo, Japan).

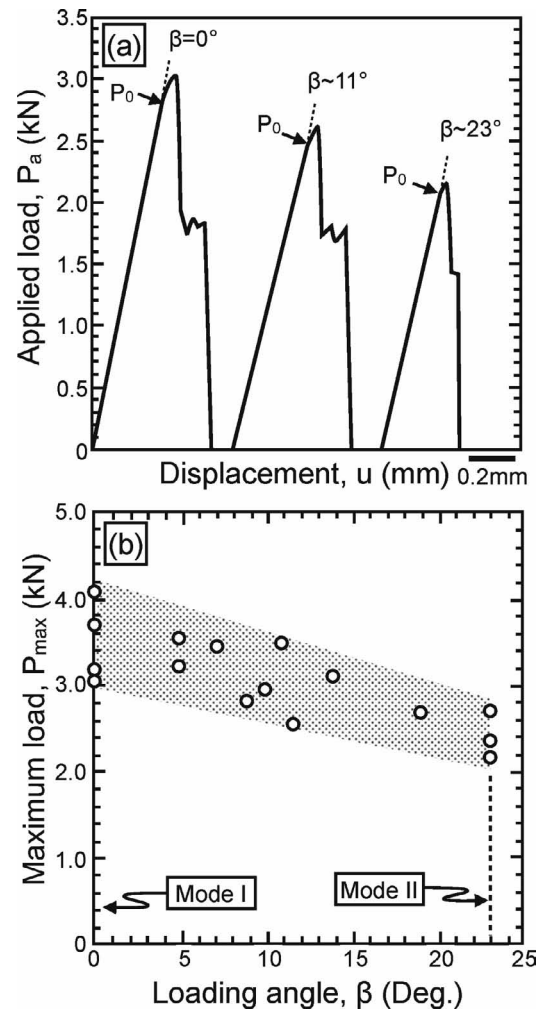


Fig. 3: (a) Typical load-displacement curves during fracture toughness test and (b) plots of the maximum load with loading angle β .

III. Experimental results

(1) Mixed-mode fracture toughness

Fig. 3a shows typical examples of applied compressive load (P_a)-displacement (u) curves as a function of the loading angle β . For all the curves, the load increases linearly from the origin to the linear proportional limit load $P_0 \approx 2.0$ – 2.8 kN. Thereafter, the curves show nonlinear behavior up to the maximum load $P_{\max} \approx 2.2$ – 4.2 kN. After reaching the maximum load, the load drops sudden-

ly to $\sim 1/2 - 2/3$ of the maximum load. Thereafter, the curve shows a short constant load stage until the specimen experiences complete failure. All load-displacement curves exhibit similar trends that are independent of the loading angles. This suggests that the load-displacement curve is independent of the loading mode. Fig. 3b shows plots of the maximum load as a function of the loading angle. The maximum load tends to decrease with increases in the angle β , *i.e.*, the load becomes smaller with the increase of the mode II component. In addition, the ratio of the nonlinear region to the entire curve becomes smaller with the increase of the loading angle β .

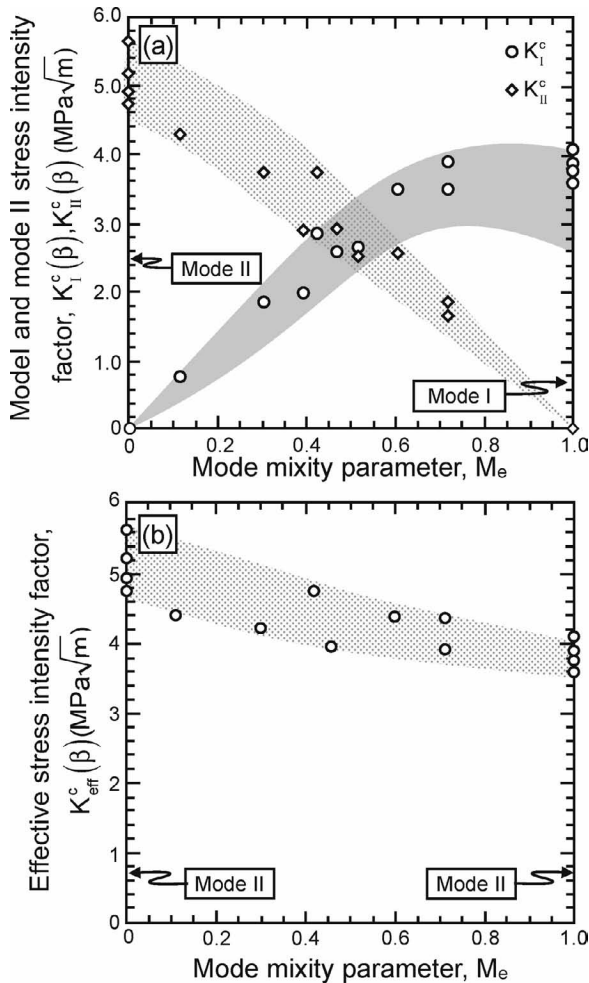


Fig. 4: (a) Plots of mode I and mode II stress intensity factor as a function of loading angle β and (b) relationship between effective stress intensity factor K_{eff}^c and mode mixity parameter M_e .

Stress intensity factors at the critical condition under various loading angles are calculated from Equations (1)–(4). Here, the maximum load was used for calculation because unstable fracture occurs at this load. Plots of the mode I and mode II stress intensity factors, $K_I^c(\beta)$ and $K_{II}^c(\beta)$, respectively, as a function of loading angles are shown in Fig. 4a. The mode I fracture toughness is measured as $K_I^c(0^\circ) = K_{IC} = 3.6 - 4.1 \text{ MPa}\sqrt{\text{m}}$, which is the same as that reported by Inoue et al.⁶ It is clear that the mode I stress intensity factor decreases with an increase of loading angle, which corresponds to an increase of the mode II component. The mode II fracture toughness is measured to be $K_{II}^c(23^\circ) = K_{IIc} = 4.7 - 5.7 \text{ MPa}\sqrt{\text{m}}$. This mode II frac-

ture toughness is $\sim 1.3 - 1.5$ larger than the mode I fracture toughness. Fig. 4b shows a plot of the effective fracture toughness $K_{\text{eff}}^c(\beta)$ vs. the mode mixity parameter M_e . The mixed-mode critical stress intensity factor increases as the mode mixity parameter decreases, and the relation always satisfies the rule $K_{\text{eff}}^c(\beta) > K_{IC}$.

(2) Macroscopic-microscopic fracture behaviors

The macroscopic fracture appearances of specimens tested under different loading modes are shown in Fig. 5. Under pure mode I loading, major crack growth starts from both of the notch tips and propagates parallel to the loading direction. Under I/II mixed-mode loading ($M_e = 0.4$) and mode II loading ($M_e = 0$), the extension of the major crack from the notch tips in the disk specimen is always non-coplanar with respect to the notch, *i.e.*, the crack always deviates from the direction of the initial notch. These observed behaviors are typical in mixed-mode fracture toughness testing of some engineering ceramics^{10–12}.

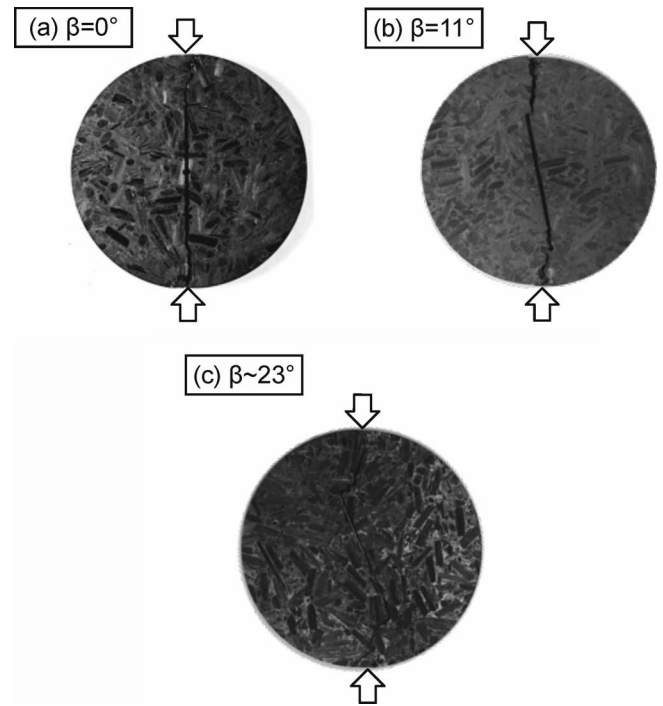


Fig. 5: Macroscopic fracture appearances of Brazilian disk specimen: (a) mode I loading ($M_e = 1$), (b) mode I+II loading ($M_e = 0.4$) and (c) mode II loading ($M_e = 0$).

Typical optical micrographs of the notch tip before and immediately after the nonlinear deformation stage for different loading modes ($M_e = 0, 0.4$, and 1) are shown in Fig. 6. As shown in Fig. 6, after the onset of non-linear deformation, many newly formed microcracks are observed near the notch tips, and most of the microcracks appear in the Si and SiC phases. This behavior is observed independent of the loading mode M_e . An example of the sequence of microcrack development under different loading conditions is shown in Fig. 7. This figure was obtained from observation of a field measuring $\sim 2 \times 2 \text{ mm}$ in front of the notch tip and the microcracks. A microcrack-induced damage zone spreads in front of the notch tip. In the mode I fracture toughness test, the size of the damage zone is $\chi_D \sim 1.8 - 2.2 \text{ mm}$. The damage zone size χ_D is

almost constant on the macroscale level and independent of the loading mode. Under mode II loading, the direction of cracks within the damage zone is perpendicular to the notch direction (parallel to the x -direction)⁶. However, the cracking directions formed under mixed-mode and mode I loading are not parallel to the x -direction. Instead, the microcracks form an angle with respect to the x -axis. The distribution of the cracking angle, which is defined as

θ_d , within the damage zone is shown in Fig. 8a. The definition of θ_d is also illustrated in Fig. 8a. The cracking angle clearly increases with the increase of the loading angle θ_d . However, the cracking angles range from 30 to 57 degrees for the case of mixed I/II mode ($M_e = 0.4$) and range from 51 to 72 degrees for the case of mode II loading. These results suggest that the loading mode strongly affects the crack formation angle in the Si/SiC phase.

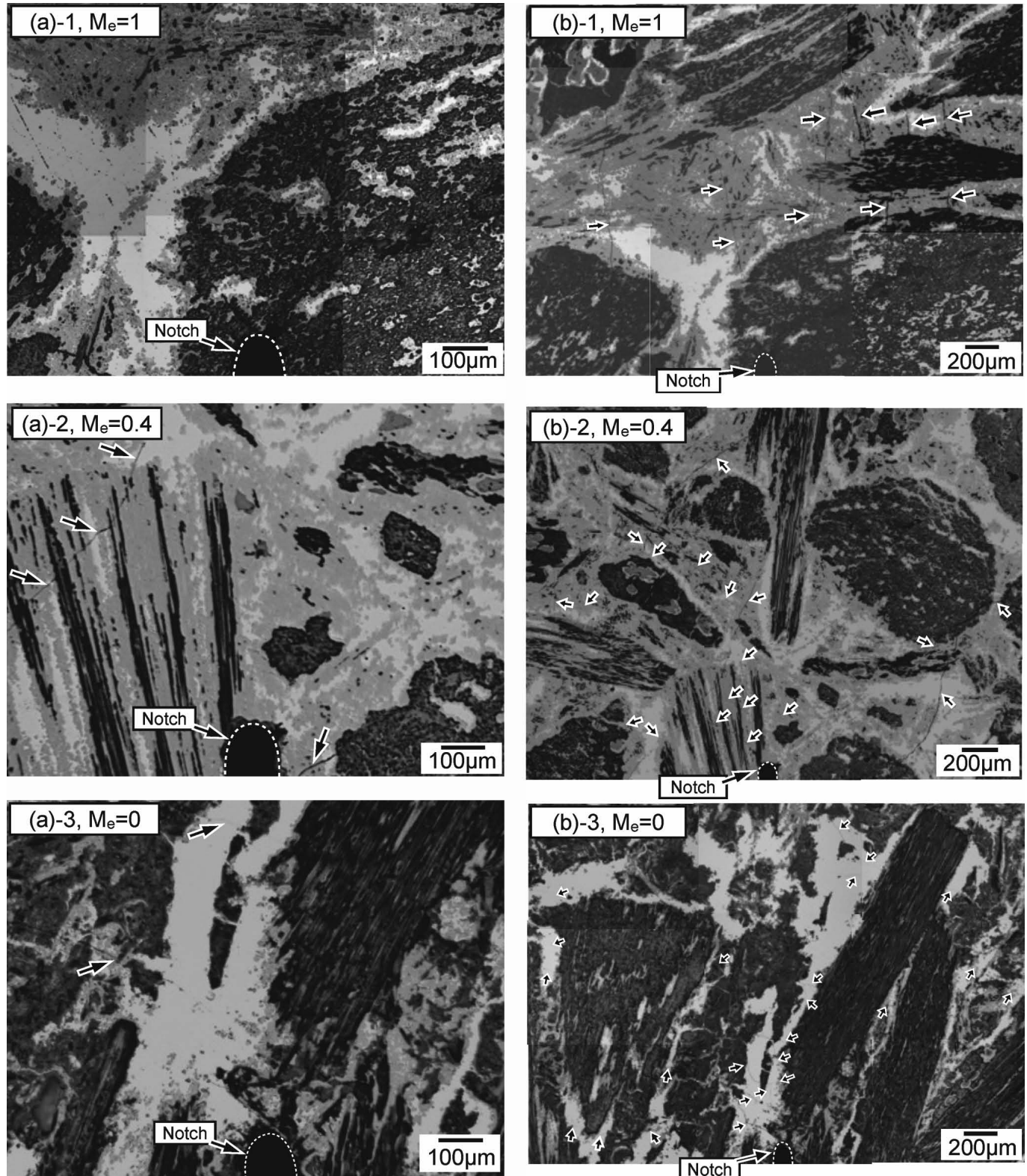


Fig. 6: Typical example of microcrack in front of notch tip: (a) before test ($M_e = 1$) and (b) at proportional limit.

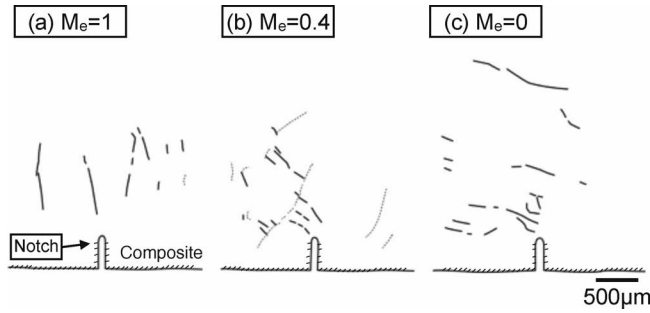


Fig. 7: Sequence of microcrack evolution: (a) mode I loading ($M_e = 1$), (b) mode I+II loading ($M_e = 0.4$), and (c) mode II loading ($M_e = 0$). Microcracks before and after test are traced as dashed and solid lines, respectively.

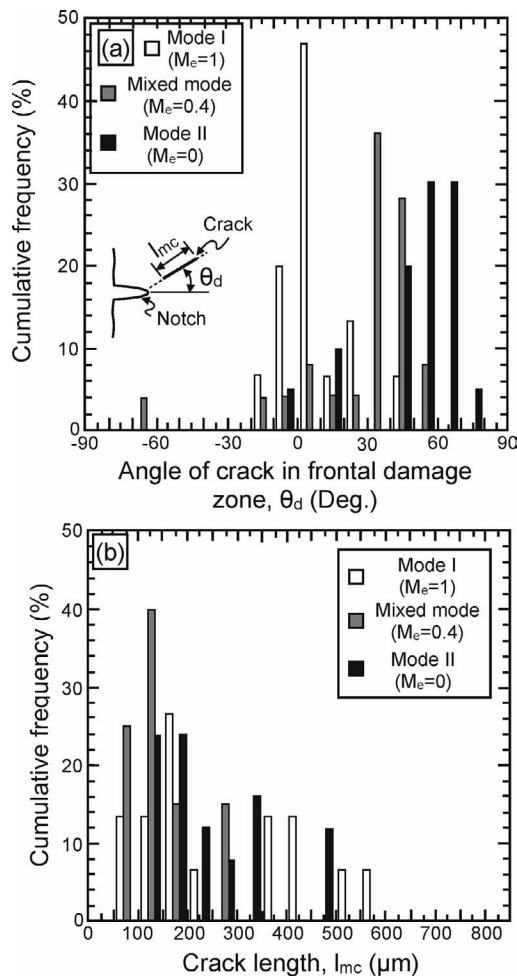


Fig. 8: Distribution of (a) crack length and (b) crack angle within damage zone in front of the notch tip, and (c) SEM image of crack-SCF/SiC mini-composite interaction.

Fig. 8b shows the distribution of microcrack length a_{mc} . All microcracks are shorter than $\sim 600 \mu m$, which is in the same range of those observed in the as-received SCF/SiC composite disk. A typical example of interactions between cracks in the damage zone and the fibers under mixed I/II mode loading is shown in Fig. 8c. The photograph (Fig. 8c) reveals that the microcrack propagation is arrested by a mini-composite phase. This phenomenon suggests that unstable long crack propagation is prevented by the SCF/SiC mini-composite phases under all applied loading modes.

IV. Discussion

Several fracture criteria under I+II mixed-mode loading have been proposed to predict unstable crack propagation in materials. These include the maximum tangential stress (hereafter denoted as MTS) criterion¹³, co-planar and non-co-planar maximum strain energy release rate criterion (hereafter denoted as G_{max} -criterion)^{14,15}, and minimum strain energy density criterion (hereafter denoted as S_{min} -criterion)¹⁶. The relationships between K_{eff}^c (β) and M_e calculated using the MTS criterion, G_{max} -criterion, and S_{min} -criterion (APPENDIX B) are shown in Fig. 9. The experimental data obtained from this study are also plotted for comparison. It is clear that the predictions based on these three well-known fracture criteria do not agree with the experimental results^{13,15,16}.

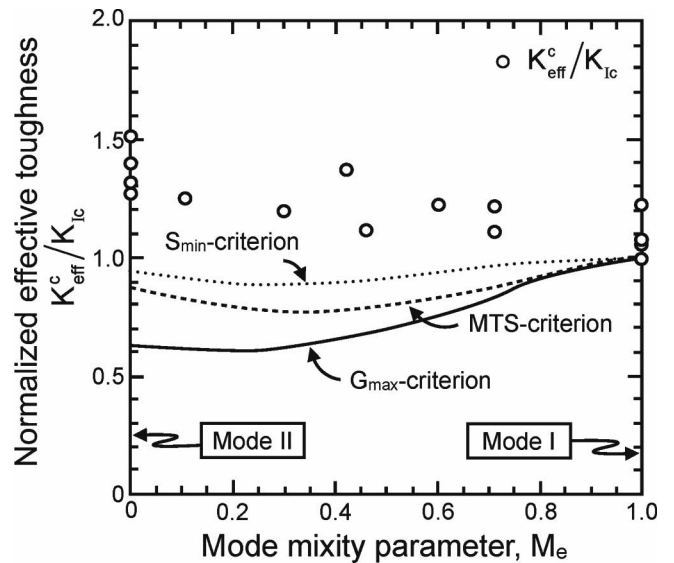


Fig. 9: Normalized effective toughness as a function of mode mixity parameter.

Deviations of experimental results from these mixed-mode fracture criteria are often observed. The deviation can be attributed to several factors, including friction between crack surfaces¹² and the effect of T -stress^{17–22}. It is especially well-known that the effect of T -stress on the mixed-mode fracture toughness is significant^{22,23}. Stress intensity parameters alone do not always enable estimation of mixed-mode fracture. The effects of higher-order T -terms in addition to singular terms must be considered²³. Previous studies have shown that compressive T -stress acts parallel to the notch direction (x -direction) in the present composite disk specimen^{20,21}. Fig. 10 shows the variations of the average strain along the x -direction

as a function of the applied load P_a measured by the strain gauge during the fracture testing. Near the origin, the compressive average strain increases linearly with the increase of applied load. Thereafter, the average strain deviates from the initial linear response and increases rapidly. At the maximum load, the average strain reaches $\sim 0.08 - 0.10\%$ for mode I loading, and $\sim 0.027 - 0.10\%$ for mode II loading. Assuming that the Young's modulus of the composite is ~ 65 GPa^{6,7}, the T -stress under mode I loading is calculated to be in the range of 48 to 60 MPa (APPENDIX C). Under mode II loading, T -stress is calculated to be from 13 to 63 MPa.

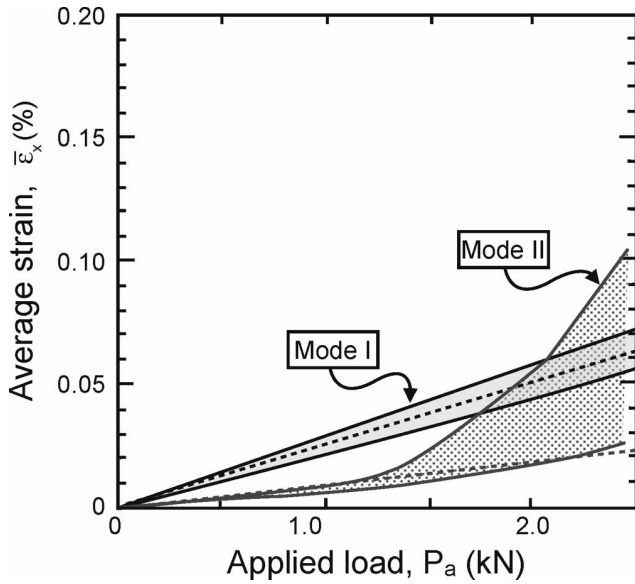


Fig. 10: Relation between average strain along x -direction and applied load.

The relationship among the stress intensity factors (K_I and K_{II}) and the T -stress (T) is given by^{18,22}:

$$K_I \sin \theta_m + K_{II} (3 \cos \theta_m - 1) - \frac{16T}{3} \sqrt{2\pi r_c} \cos \theta_m \sin \left(\frac{\theta_m}{2} \right) = 0, \quad (7)$$

where θ_m is the fracture propagation direction from the crack and r_c is the critical distance from the crack tip. T is a constant and non-singular term independent of the distance from the crack tip. From this equation, the critical distance r_c can be obtained using θ_m and θ_d . The generalized MTS criterion is given by^{18,22}:

$$\frac{K_{eff}^c}{K_{Ic}} = \frac{\sqrt{Y_I^2 + Y_{II}^2}}{Y_I \cos^3 \left(\frac{\theta_m}{2} \right) - 3Y_{II} \sin \theta_m \cos \left(\frac{\theta_m}{2} \right) + \frac{RT^*}{R - a_0} \sqrt{\frac{2r_c}{a_0}} \sin 2\theta_m}, \quad (8)$$

where T^* is the normalized T -stress, which is given by^{20,21}:

$$\frac{T^*}{T} = \frac{\pi t(R - a_0)}{P_a}. \quad (9)$$

The shape factors, Y_I and Y_{II} , are calculated from Eqs. (3) and (4). From Eq. (7), the critical distance r_c is calculated to be within the range of 0.5 mm to 3.8 mm. Although the calculated critical distance has a wide range of scattering, it is almost comparable with the size of the observed damage zone formed in front of the notch tip (cf. Fig. 7).

Fig. 11a shows the relationship between the measured cracking angle within the damage zone and the mode mixity parameter M_e . The dashed line denotes the calculated result using Eq. (7). The results show that the measured crack angles are always larger than those calculated from the MTS-criterion (APPENDIX B). This result is in agreement with previous analysis^{20,21} and suggests that compressive T -stress is important in determining the fracture criterion in the present case. Cracking angles in the damage zone predicted based on Eq. (8) with $r_c = 0.5$ mm and 3.8 mm are shown by solid lines in Fig. 11a. Fig. 11b also shows the relationship between mixed-mode fracture toughness and the mode mixity parameter using the plots calculated from Eqs. (8) and (9). Compared among the three fracture criteria, the prediction based on the generalized MTS criterion shows the best agreement with the experimental data for the SCF/SiC composite. This result confirms that compressive T -stress is crucial in determining the mixed-mode fracture toughness of the SCF/SiC composite under the present experimental conditions.

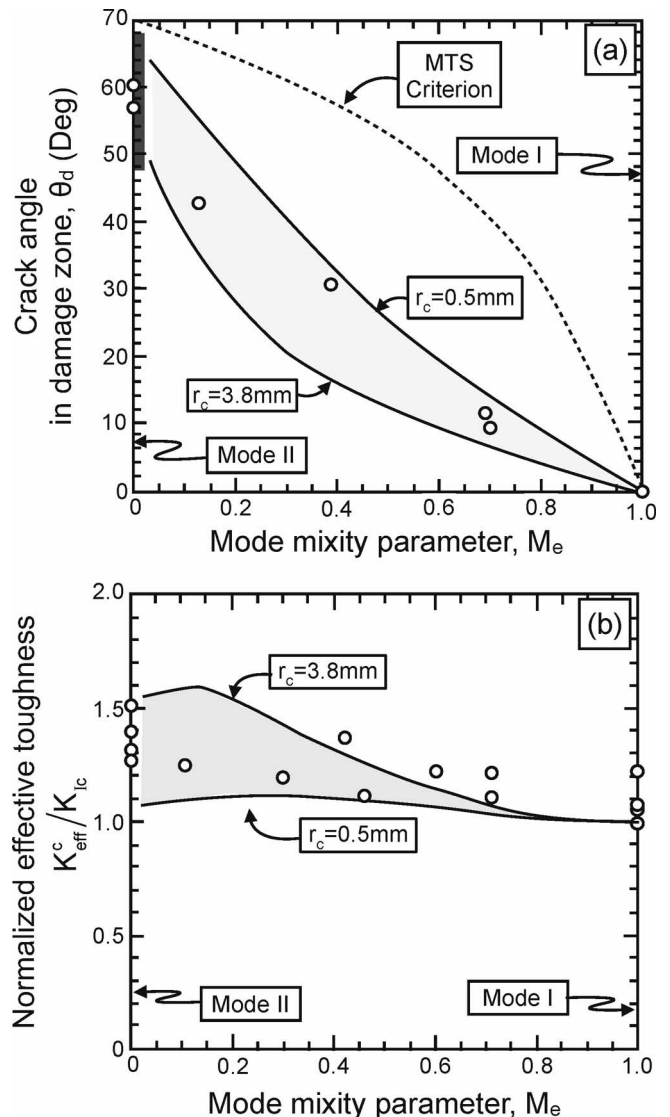


Fig. 11: Prediction of mixed-mode fracture criterion of SCF/SiC composite. (a) θ_d as a function of mode mixity parameter, and (b) K_{eff}^c / K_{Ic} as a function of mode mixity parameter.

V. Conclusions

Fracture behavior and toughness of an in-plane randomly oriented SCF/SiC composite under pure and mixed-mode loading (mode I and mode II) were examined using a Brazilian disk specimen.

(1) Mode I and mode II fracture toughness values of the DCF/SiC composite are $K_{Ic} \sim 3.6 - 4.1 \text{ MPa}\sqrt{\text{m}}$ and $K_{IIc} = 4.7 - 5.7 \text{ MPa}\sqrt{\text{m}}$. The mode II fracture toughness is ~ 1.5 times larger than mode I fracture toughness. These fracture toughness values are in the same range as those of monolithic engineering ceramics.

(2) The damage zone in front of the notch tip was observed under different applied loading mode conditions. Under mixed I/II mode and pure mode II loading, the microcrack orientations within the damage zone are different from those under pure mode I loading.

(3) The angle between the notch direction and crack propagation direction varied depending on the mode. The K_{eff} - M_e relation deviated from fracture criteria based on MTS, G_{max} , and S_{min} . Compressive stress acted parallel to the notch direction. The fracture criterion considering the influence of this compressive T -stress agrees well with the experimental data obtained from the mixed-mode fracture toughness test for the SCF/SiC composite, showing that the T -stress plays an important role in the present material system.

Appendix A

The dimensionless coefficients needed for Eqs. (3) and (4) are listed in Table A-1 and Table A-2. These parameters are obtained from reference ⁸.

Table A-1: First five coefficients for Eqs. (3) and (4).

T_1	1.387239	S_1	1.257488
T_2	0.594892	S_2	0.563966
T_3	0.421949	S_3	0.406869
T_4	0.428353	S_4	0.410966
T_5	0.347941	S_5	0.336447

Table A-2: First five angular constants for Eqs. (3) and (4).

A_1	$1-4\sin^2\beta$	B_1	1
A_2	$8\sin^2\beta(1-4\cos^2\beta)$	B_2	$-5+8\cos^2\beta$
A_3	$-4\sin^2\beta$ $(3-36\cos^2\beta+4\cos^4\beta)$	B_3	$-3+16(1-2\cos^2\beta)(2-3\cos^2\beta)$
A_4	$-16\sin^2\beta$ $(-1+24\cos^2\beta-80\cos^4\beta+64\cos^6\beta)$	B_4	$3+16(1-2\cos^2\beta)$ $-12(1-2\cos^2\beta)^2-32(1-2\cos^2\beta)^3$
A_5	$-20\sin^2\beta$ $(1-40\cos^2\beta+240\cos^4\beta$ $-448\cos^6\beta+256\cos^8\beta)$	B_5	$5-16(1-2\cos^2\beta)$ $-60(1-2\cos^2\beta)^2+32(1-2\cos^2\beta)^3$ $+80(1-2\cos^2\beta)^4$

Appendix B

The tangential stress $\sigma_{\theta\theta}$ in front of the crack tip can be given as an infinite-series expansion expressed using the mode I and mode II stress intensity factors ²³:

$$\sigma_{\theta\theta} = \frac{1}{\sqrt{2\pi r_c}} \cos\left(\frac{\theta}{2}\right) \left[K_I \cos^2\left(\frac{\theta}{2}\right) - \frac{3}{2} K_{II} \sin\theta \right] + T \sin^2\theta + O(r^{1/2}) \quad (\text{B-1})$$

where r and θ are given as in Fig. 2b. T and $O(r^{1/2})$, respectively, represent the T -stress and the remaining higher-order terms of the series expansion, which are negligible near the crack tip. The MTS criterion assumes that fracture occurs in the direction perpendicular to the MTS direction. In the case that T -stress is negligible, the fracture condition is given by ¹³:

$$K_{Ic} = \cos\left(\frac{\theta_m}{2}\right) \left[K_I \cos^2\left(\frac{\theta_m}{2}\right) - \frac{3}{2} K_{II} \sin\theta_m \right] \quad (\text{B-2})$$

where θ_m is the fracture initiation angle. θ_m is given by:

$$\cos\left(\frac{\theta_m}{2}\right) \cdot [K_I \sin\theta_m + K_{II}(3\cos\theta_m - 1)] = 0. \quad (\text{B-3})$$

Although the angle usually changes during crack propagation, the angle just after crack growth from the notch tip is defined as θ_m .

The G_{max} criterion assumes that crack extension occurs in the initial crack direction when G reaches the mode I critical strain energy release rate. The fracture condition is given in the following relationship ¹⁵:

$$K_{Ic} = \left\{ b_{11}(\theta_m) K_I^2 + b_{12}(\theta_m) K_I K_{II} + b_{22}(\theta_m) K_{II}^2 \right\}^{1/2} \cdot \left\{ \frac{(1-\theta_m)/\pi}{(1+\theta_m)/\pi} \right\}^{\theta_m/\pi} \quad (\text{B-4})$$

where

$$\begin{aligned} b_{11}(\theta_m) &= \frac{4(1+3\cos^2\theta_m)}{(3+\cos^2\theta_m)^2}, \\ b_{12}(\theta_m) &= \frac{32\sin\theta_m\cos\theta_m}{(3+\cos^2\theta_m)^2}, \end{aligned} \quad (\text{B-5})$$

and

$$b_{22}(\theta_m) = \frac{4(9-5\cos^2\theta_m)}{(3+\cos^2\theta_m)^2}.$$

A fracture criterion based on S_{min} is also proposed. In this criterion, the crack propagates in the direction in which the strain energy density is minimal. The S_{min} criterion is given by 16:

$$K_{Ic} = \frac{1}{2(\kappa-1)} a_{11} K_I^2 + 2a_{12} K_I K_{II} + 2a_{22} K_{II}^2, \quad (B-6)$$

where

$$\begin{aligned} a_{11} &= (1+\cos\theta_m)(\kappa-\cos\theta_m), \\ a_{12} &= \sin\theta_m(2\cos\theta_m-\kappa+1), \\ \text{and} \\ a_{22} &= (\kappa+1)(1-\cos\theta_m)+(1+\cos\theta_m)(3\cos\theta_m-1), \end{aligned} \quad (B-7)$$

where ν is Poisson's ratio, with $\kappa = 3-4\nu$ in the plane strain condition. In the present study, $\nu = 0.2$ is used for calculation (Table 1).

APPENDIX C

The stress component along the x -direction $\sigma_x(r, \theta)$ under mode I loading condition is given by 23:

$$\sigma_x(r, \theta) \approx \frac{1}{\sqrt{2\pi r}} \{K_I f(\theta) + K_{II} g(\theta)\} + T, \quad (C-1)$$

where $f(\theta)$ and $g(\theta)$ are given by 23:

$$f(\theta) = \cos\left(\frac{\theta}{2}\right) \left\{1 - \sin\left(\frac{\theta}{2}\right) \sin\left(\frac{3\theta}{2}\right)\right\}, \quad (C-2)$$

and

$$g(\theta) = -\sin\left(\frac{\theta}{2}\right) \left\{2 + \cos\left(\frac{\theta}{2}\right) \cos\left(\frac{3\theta}{2}\right)\right\}. \quad (C-3)$$

The average stress in the region bounded by the strain gauge $\bar{\sigma}_x$ in the present test condition is:

$$\bar{\sigma}_x \sim \frac{1}{A} \int_{r_1}^{r_1+d} \int_{-\theta}^{\theta} \sigma_x(x, \theta) dx d\theta, \quad (C-4)$$

where A is the area of the strain gauge. Eq. (C-4) becomes:

$$\begin{aligned} \bar{\sigma}_x \sim \frac{1}{A} \left[K_I \left\{ 3 \sin\left(\frac{d\theta}{2}\right) + \frac{1}{5} \sin\left(\frac{5d\theta}{2}\right) \right\} + 2Td\theta \right] \\ \cdot \sqrt{\frac{2}{\pi}} \left(\sqrt{r_1+d} - \sqrt{r_1} \right). \end{aligned} \quad (C-5)$$

The T -stress (T) is given by combining Eqs. (C-1)–(C-6):

$$\begin{aligned} T = 2d\theta A \bar{E}_x \bar{E}_x \left\{ \sqrt{\frac{2}{\pi}} \left(\sqrt{r_1+d} - \sqrt{r_1} \right) \right\}^{-1} \\ - \frac{1}{2d\theta} \left[K_I \left\{ 3 \sin\left(\frac{d\theta}{2}\right) + \frac{1}{5} \sin\left(\frac{5d\theta}{2}\right) \right\} \right]. \end{aligned} \quad (C-6)$$

$\bar{E}_x^L$ is the Young's modulus along the x -direction. The area of the strain gauge and the angle are $A \sim 0.5 \text{ mm}^2$, $d \sim 0.5 \text{ mm}$. In the area of (C-1)–(C-6), the criterion given by 6, 7) is used for estimation. T -stress near the notch tip is roughly estimated to be 48–60 MPa under mode I loading and 13–63 MPa under mode II loading. In the present test conditions, the T -stress under the mode I loading condition is similar to that under the mode II loading condition.

References

- 1 Krenkel, W., Carbon Fiber Reinforced Silicon Carbide Composites (C/SiC, C/C-SiC), In: Handbook of ceramic composites, Springer, US 117–148, (2005).
- 2 Krenkel, W., Heidenreich, B., Renz, R.: C/C-SiC Composites for advanced friction systems, *Adv. Eng. Mater.*, **4**, 427–436, (2002).
- 3 Krenkel, W.: Carbon fiber reinforced CMC for high-performance structures, *Int. J. Appl. Ceram. Tech.*, **1**, [2] 188–200, (2004).
- 4 Krenkel, W., Berndt, F.: C/C-SiC composites for space applications and advanced friction systems, *Mater. Sci. Eng. A*, **412**, 177–181, (2005).
- 5 El-Hija, H. A., Krenkel, W., Hugel, S.: Development of C/C-SiC brake pads for high-performance elevators, *Int. J. Appl. Ceram. Tech.*, **2**, 105–113, (2005).
- 6 Inoue, R., Yang, J. M., Kakisawa, H., Kagawa, Y.: Mode I fracture toughness of short carbon fiber-dispersed SiC matrix composite fabricated by melt infiltration process, *Ceram. Int.*, **39**, [7], 8341–8346, (2013).
- 7 Inoue, R., Kakisawa, H. and Kagawa, Y.: Fracture criterion of short carbon fiber-dispersed SiC matrix composite under mixed mode loading condition. In: Design, development, and applications of structural ceramics, composites, and nanomaterials: Ceramic transactions, Vol. 244 (Eds D. Singh, D. Zhu, W. M. Kriven, S. Mathur and H.-T. Lin), John Wiley & Sons, Inc., Hoboken, NJ, USA, (2014).
- 8 Atkinson, C., Smelser, R. E., Sanchez, J.: Combined mode fracture via the cracked brazilian disk test, *Int. J. Fract.*, **18**, [4], (1982).
- 9 Shih, C. F.: Small-scale yielding analysis of mixed mode plane-strain crack problems. In: Fracture Analysis: Proceedings of the 1973 National Symposium on Fracture Mechanics, Part II. ASTM International, 1974.
- 10 Sherry, D. K., Rosenfield, A. R.: Slow crack growth in glass in combined mode I and mode II loading, *Scripta. Mater.*, **25**, 997–1002, (1991).
- 11 Shetty, D. K.: Mixed-Mode Fracture in Biaxial Stress State: Application of the Diametral-Compression (Brazilian Disk) Test, *Eng. Fract. Mech.*, **26**, 6, 825–840, (1987).
- 12 Singh, D., Shetty, D. K.: Fracture toughness of polycrystalline ceramics in combined mode I and mode II loading, *J. Am. Ceram. Soc.*, **12**, [1] 78–84, (1989).
- 13 Erdogan, F., Sih, G. C.: On the crack extension in plates under plane loading and transverse shear, *J. Basic. Eng.-T ASME*, **85**, 519–527, (1963).
- 14 Hussain, M. A., Pu S. L., Underwood, J.: Strain energy release rate for a crack under combined mode I and mode II. In: *Fracture Analysis: Proceedings of the 1973 National Symposium on Fracture Mechanics, Part II*. ASTM International, 1974.
- 15 Palaniswamy, K. K. W. G., Knauss, W. G.: On the problem of crack extension in brittle solids under general loading, *Mechanics today* **4**, [30], 87–148, (1978).
- 16 Sih, G. C.: Strain-energy-density factor applied to mixed mode crack problems, *Int. J. Fract.*, **10**, 305–321, (1974).
- 17 Williams, J. G., Ewing, P. D.: Fracture under complex stress-the angled crack problem, *Int. J. Fract.*, **8**, 441–416, (1972).
- 18 Smith, D. J., Ayatollahi, M. R., Pavier, M. J.: On the consequences of T -stress in elastic brittle fracture, *Proc. R. Soc. Lond.*, **462**, [2072], 2415–2437, (2006).
- 19 Ayatollahi, M. R., Aliha, M. R. M.: Fracture analysis of some ceramics under mixed mode loading, *J. Am. Ceram. Soc.*, **94**, [2], 561–569, (2011).
- 20 Ayatollahi, M. R., Aliha, M. R. M.: On the use of brazilian disc specimen for calculating mixed mode I-II fracture toughness of rock materials, *Eng. Fract. Mech.*, **75**, [16], 4631–4641, (2008).

- ²¹ Fett, T.: Stress intensity factors and T-stress for internally cracked circular disks under various boundary conditions, *Eng. Fract. Mech.*, **68**, [9], 1119–1136, (2001).
- ²² Smith, D.J., Ayatollahi, M.R., Pavier, M.J.: The role of T-stress in brittle fracture for linear elastic materials under mixed-mode loading, *Fatigue Fract. Eng. Mater. Struct.*, **24**, [2], 137–150, (2001).
- ²³ Williams, M.L.: On the stress distribution at the base of a stationary crack, *J. Appl. Mech.*, **24**, 109–114, (1957).